UNIFIED ANALYSIS OF QUASI-TEM AND HIGHER ORDER MODES IN PLANAR TRANSMISSION LINES

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Abstract

TEM modes are shown to be derivable from a scalar magnetic potential ψ if the supporting transmission line can be considered a slot coupled waveguide with more than one coupling slot. The magnetic potential must jump at the coupling slots in order to give rise to the axial electric current on the strips separating these slots. The advantage of this formulation is its compatibility to the numerically efficient generalized spectral domain (GSD) technique which has already been used for the calculation of TE and TM modes in slot-coupled waveguides. Substrates are taken into account by applying the eigenmode transformation technique. Numerical results are presented for TEM modes in multi-slot lines and for quasi-TEM and higher order modes in coplanar waveguides. Excellent agreement with the results obtained by other methods have been achieved with moderate cpu time and storage requirements.

Introduction

The numerically efficient generalized spectral domain (GSD) technique is the generalization of the conventional one-dimensional spectral domain technique [1],[2]. It has been applied to the analysis of both guiding and resonance structures (see e.g. [3]–[5]). This technique is based on subdividing the structure to be analyzed into well-defined regions which are coupled by apertures. The latter are next short-circuited and the non-vanishing tangential electric field there is restored by inserting surface magnetic currents at both sides of the introduced short circuit. This procedure has the advantage of separating the structure into subregions which are independently analyzed. This gives the method its modular feature.

According to the method described in [5] TEM modes are in fact TE modes with a vanishing cutoff wavenumber. In order to include the analysis of the TEM modes in the GSD technique, it is necessary to describe these modes in terms of a scalar magnetic rather than the conventional electric potential. The existence of the axial electric current enforces the magnetic potential to be multi-valued, which gives rise to numerical difficulties. In this contribution it is shown that an equivalent alternative to the multi-valued feature in a class of slot-coupled transmission lines is to let the magnetic potential jump across the coupling apertures. Except there the magnetic potential remains well-behaved and single-valued.

Basic Formulation

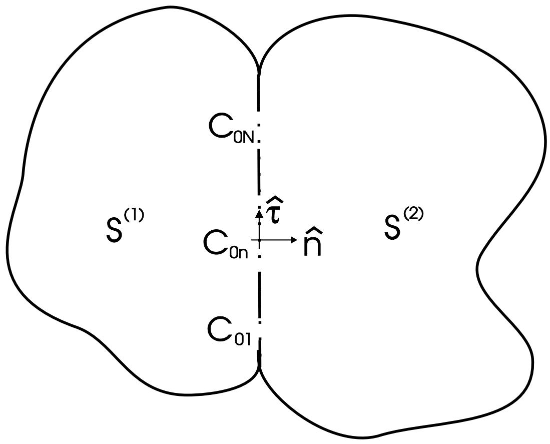

Fig. 1: N-slot-coupled waveguides

First TEM modes in waveguides which can be decomposed into two or more slot-coupled waveguides are considered. Because the composite structure is still a waveguide, the coupling slots must be axially uniform. We will consider the case where two adjacent waveguides are coupled by two or more slots so that at least one TEM mode of the composite structure exists. Consider the composite waveguide with the cross

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section shown in Fig. 1. The two waveguides (1) and (2) with cross sections $S^{(1)}$ and $S^{(2)}$, respectively, are slot-coupled through the N slots C_{0n} $(n = 1, 2, \ldots, N)$, which may be combined to a single composite contour C_0. The N slots correspond to $(N-1)$ strips which are isolated from the shielding waveguide, so that $(N-1)$ TEM modes can be supported by the composite waveguide.

Let $\boldsymbol{e}$ and $\boldsymbol{h}$ be the transverse electric and transverse magnetic field, respectively. Short-circuiting the composite coupling slot C_0 we can follow the basic formulation given in [5] which gives rise to equivalent axial magnetic currents $\pm m_{sz}$ to the left (right) side of the short circuit, respectively. All field components in waveguide (i) can be expressed in terms of a scalar magnetic potential $\psi^{(i)}$ as follows:

$$\boldsymbol{h}^{(i)} = -\boldsymbol{\nabla}_t \psi^{(i)} \quad , \tag{1}$$

$\psi^{(i)}$ is readily shown to satisfy the following differential equation:

$$\boldsymbol{\nabla}_t^2 \psi^{(i)} = \mp \frac{1}{Z_0} m_{sz} \delta(n - n_0) \tag{2}$$

with a Neumann's boundary condition.

Due to the continuity of the tangential magnetic field at C_0 the tangential derivates of $\psi^{(i)}$ must be continuous across the slots. However, there is no need to force the continuity of the magnetic potential $\psi^{(i)}$ at C_0. The difference between $\psi^{(1)}$ and $\psi^{(2)}$ at C_{0n} must then be constant along C_{0n}. A jump in ψ across C_{0n} does not lead to a dirac-delta behaviour for the normal magnetic field $h_n = \partial\psi/\partial n$, because (1) is valid for the individual waveguides only, but not for the composite waveguide. This can be explained by the fact that $\boldsymbol{h}^{(i)}$ is curl-free in each individual waveguide. On the other hand, if we have more than one slot, an axial electric current will exist on the strips between them giving rise to a non vanishing $\boldsymbol{\nabla}_t \times \boldsymbol{h}$ in the composite waveguide. In fact jumps in $\psi^{(i)}$ across the coupling slots are necessay for a TEM mode in case of more than one slot.

Let us now define the jump in $\psi^{(i)}$ across C_{0n} at the nth slot as an independent variable:

$$\Delta\psi_n = \psi^{(2)}(P_n) - \psi^{(1)}(P_n) \quad , \tag{3}$$

where P_n is an arbitrary point of C_{0n} $(1 \le n \le N)$. (Note that such a jump is constant along the nth slot.) The N slots correspond to exactly $(N-1)$ linearly independent TEM solutions. One possible choice of linearly independent TEM modes is the following : Let the nth TEM mode correspond to the case that the nth strip has an electric potential of unity with respect to the grounded shielding waveguide, while all other $(N-2)$ strips are grounded. Integrating the magnetic current along the kth slot results in

$$\int_{C_{0k}} m_{sz}\, dl = \delta_{k,n} - \delta_{k,n+1} \quad , \tag{4}$$

where $\delta_{k,n}$ is the Kronecker delta.

Due to the Neumann's boundary condition let us expand the magnetic potential $\psi^{(i)}$ in each individual waveguide in terms of the complete set $\left\{h_{zn}^{(i)}\right\}$, describing the axial magnetic fields of TE modes in the individual waveguides:

$$\psi^{(i)} = \sum_{\mu}^{\infty} a_\mu^{(i)} h_{z\mu}^{(i)} \quad . \tag{5}$$

The method of moments is now applied by expanding the axial surface magnetic current m_{sz} at the nth slot in terms of suitably chosen basis functions $m_{zk}^{(n)}$ which should satisfy the edge conditions [6] in order to reduce their number and hence the computational efforts. After some mathematical manipulations one arrives at the following matrix equation which relates the field expansion coefficients $\boldsymbol{a}^{(i)}$ with the expansion coefficients of the magnetic current $\boldsymbol{b^{(n)}}$.

$$\boldsymbol{a}^{(i)} = \pm \sum_{n=1}^{N} \left[R_n^{(i)}\right] \boldsymbol{b^{(n)}} \quad , \tag{6}$$

The upper (lower) sign corresponds to $i = 1$ $(i = 2)$, respectively. The elements of $\left[R_n^{(i)}\right]$ are given by

$$(R_n^{(i)})_{\mu k} = \int_{C_{0n}} h_{z\mu}^{(i)} m_{zk}^{(n)}\, dl \quad . \tag{7}$$

Applying Galerkin's procedure and making use of (4) leads to the characteristic equation for the nth TEM mode $(1 \le n \le N-1)$:

$$\begin{bmatrix} [C_{11}] & [C_{12}] & \cdots & [C_{1N}] & \boldsymbol{V_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ [C_{21}] & [C_{22}] & \cdots & [C_{2N}] & \boldsymbol{0} & \boldsymbol{V_2} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ [C_{N1}] & [C_{N2}] & \cdots & [C_{NN}] & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{V_N} \\ \boldsymbol{V_1}^t & \boldsymbol{0}^t & \cdots & \boldsymbol{0}^t & 0 & 0 & \cdots & 0 \\ \boldsymbol{0}^t & \boldsymbol{V_2}^t & \cdots & \boldsymbol{0}^t & 0 & 0 & & 0 \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \boldsymbol{0}^t & \boldsymbol{0}^t & \cdots & \boldsymbol{V_N}^t & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{b^{(1)}} \\ \boldsymbol{b^{(2)}} \\ \vdots \\ \boldsymbol{b^{(N)}} \\ \Delta\psi_1 \\ \vdots \\ \Delta\psi_n \\ \Delta\psi_{n+1} \\ \vdots \\ \Delta\psi_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \\ 0 \\ \vdots \\ 1 \\ -1 \\ \vdots \\ 0 \end{bmatrix} , \tag{8}$$

where the matrices $[C_{ik}]$ are defined by

$$[C_{ik}] = \left[R_i^{(1)}\right]^t \left[R_k^{(1)}\right] + \left[R_i^{(2)}\right]^t \left[R_k^{(2)}\right] \quad . \tag{9}$$

and the column vector $\boldsymbol{V_n}$ contains the elements

$$V_k^{(n)} = \int_{C_{0n}} m_{zk}^{(n)} \, dl \quad . \tag{10}$$

It is to be noted that for the elements $[C_{ik}]$ of the characteristic matrix and for the evaluation of the field series, doubly infinite sums over the eigenmodes of the individual waveguides have to be evaluated. The efficiency of the GSD method can considerably be enhanced if those sums normal to the slots are replaced by closed-form expressions, as has been described in [7]. This reduces the computational cpu time and storage requirements to the extent that the method is easily integrated in any optimization routine.

Many planar transmission lines, e.g. coplanar and coupled microstrip lines, contain a dielectric substrate to support the conducting strips. The effect of arbitrary inhomogeneous substrates can be considered by applying the eigenmode transformation technique [8]. For an N-slot-coupled transmission line the electromagnetic fields are expanded in terms of the TE , TM and $(N-1)$ TEM modes. In [5] it has been derived how TE and TM modes can be calculated. The computation of TEM modes by the GSD technique is part a of this contribution, which permits a unified description for all expansion modes. Applying the eigenmode transformation technique results in a proper eigenvalue problem whose eigenvalues are the normalized propagation contstants squared $(\beta/k_0)^2$ (k_0 denotes the free space wavenumber) of the inhomogeneously filled waveguide and whose eigenvectors contain the expansion coefficients of the transverse electric field. In general the modes of the inhomogeneously filled waveguide are hybrid and for a multi-slot-coupled transmission line the dominant modes are quasi-TEM.

Numerical Results

In order to validate the suggested method a coplanar slotline with three slots is first analyzed. This structure supports two TEM modes. In Fig. 2 the electric and magnetic field lines are presented for both TEM modes which have been defined by having a potential of unity on one of the two strips, and a zero potential on the other one. It is worth noting that the two TEM modes are not orthogonal but by choosing suitable linear combinations of these modes an orthogonal set of TEM modes can be defined.

In Fig. 3 the axial surface magnetic current (solid line) is compared with the field component E_y in the plane of the slots (dashed line) for both TEM modes, which must be equal according to the definition of the surface magnetic current. This is in good agreement with the plots.

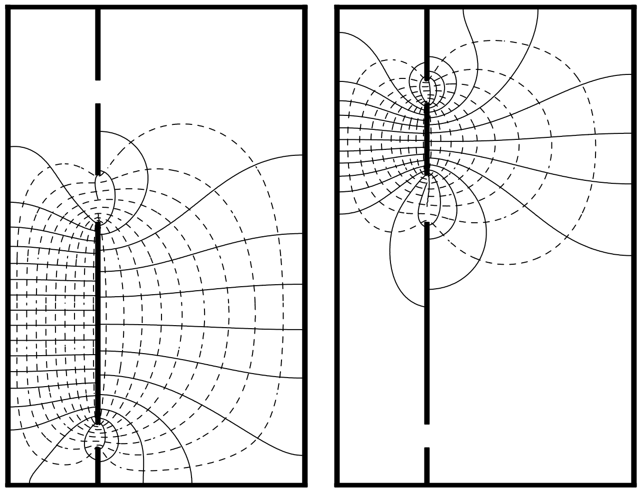

Fig. 2: Electric (—) and magnetic (- -) field lines for the two TEM modes of a three-slot coplanar slot line

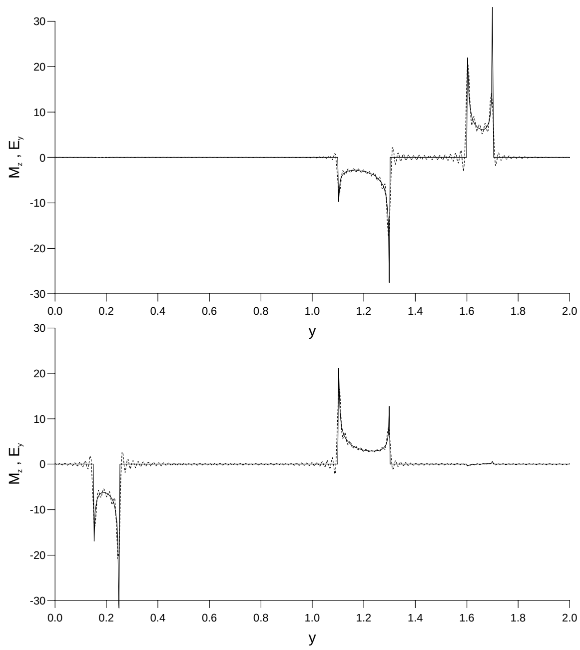

Fig. 3: Comparison between the slot tangential electric fields (- -) and the axial surface magnetic current (—) corresponding to the two TEM modes.

Next we consider a structure with a dielectric substrate. Fig. 4 shows the cross section of a shielded coplanar waveguide. Fig. 5 presents the results of the eigenmode transformation technique. The dispersion characteristic of the quasi-TEM and some higher modes of a coplanar waveguide are shown there. Excellent agreement has been obtained with the results of [9], which were calculated by a full-wave integral equation method. On the other hand, the presented method can accurately determine as many modes as one needs for the analysis of, e.g., discontinuities with moderate cpu time and storage requirements.

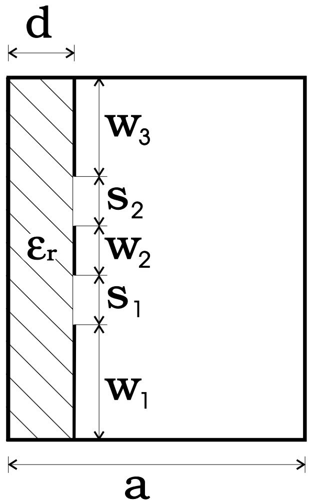

Fig. 4: Cross section of a coplanar waveguide.

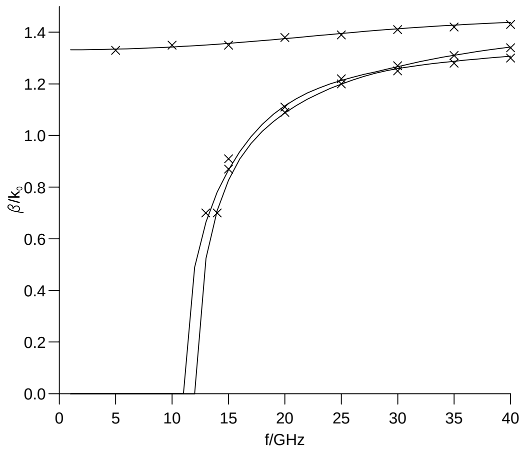

Fig. 5: Mode spectra of the quasi-TEM and higher order modes of the CPW shown in Fig. 4; $s_1 = s_2 = 0.381\,\text{mm}$, $w_1 = 3.54\,\text{mm}$, $w_2 = 2.032\,\text{mm}$, $w_3 = 3.666\,\text{mm}$, $d = 0.7874\,\text{mm}$, $a = 10\,\text{mm}$, $\epsilon_r = 2.2$; $\times$: results of [9].

Conclusions

A new method for the computation of TEM modes in multi-slot transmission lines has been derived. The supporting transmission line has been treated as a slot-coupled waveguide with more than one coupling slot. Applying the GSD technique, all field components have been expressed in terms of a scalar magnetic potential that must jump at the coupling slots. Field plots of TEM modes in multi-slot lines have been presented, and it has been shown how these modes can be used to compute quasi-TEM and higher order modes in coplanar waveguides.

Acknowledgement

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